IOWA STATE UNIVERSITY | Debbie and Jerry Ivy College of Business

Bayesian Analysis: A Conceptual Introduction

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Research Methods Division
Professional Development Workshop

Bayesian Statistics:
How to Conduct and Publish High-Quality Bayesian Studies

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ACADEMY OF MANAGEMENT

2011, 2012, 2013, 2014, 2015, 2017, 2018, 2019, 2020, 2022, 2023

History of Bayesian Analysis

- Old and established

Reverend Thomas Bayes
7 April 1761 (aged 59)

History of Bayesian Analysis

- WWII
- Established tool in applied statistics
- Medical, Engineering, Marketing and other fields
- Machine-learning and AI draw on Bayesian analyses

Famous decoding of Engima code during WWII

McGrayne (2011). The Theory That Would Not Die.

Webcast Objective

- Conceptual introduction to Bayesian analysis
- Bayesian is a complex and flexible statistical tool
- Impossible to cover all aspects in one webcast
- Focus on “big picture issues”
- Less focus on technical issues of executing and evaluating Bayesian results.
- … but will provide some references and ideas on how to obtain needed additional training.

Webcast Structure

(1) Logic of Bayesian Analysis: Bayes' Theorem

(2) Estimating Posterior Distributions

(3) Role and Specification of Priors Distributions

(4) Execution of Bayesian Estimation

(5) Reporting Results and Publishing Bayesian Studies in Management Journals

Logic of Bayesian Analysis: Bayes' Theorem

Logic of Bayesian Analysis

- Statistical significance tests answers
  - Probability of observing data due to random sampling assuming null-hypothesis is true

    $Pr(data \mid H_0)$ (no probability statements for H_1 or H_0 being true)
- The real question of interest
  - Probability of proposed hypothesis being true given the observed data

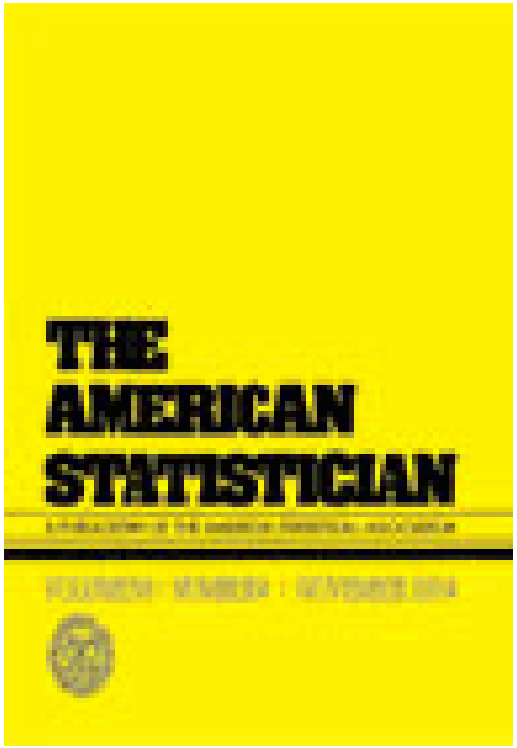
    $Pr(H_1 \mid data)$
- Bayesian analyses try to answer this question!

Need to move beyond statistical significance and p < .05

American Statistical Association

"Statement on p-Values" (2016)

"Moving beyond p < .05" (2019)



Wasserstein, R. L., & Lazar, N. A. 2016. The ASA's statement on p-values: context, process, and purpose. ***The American Statistician***, 70(2): 129-133.

Wasserstein, R.L., Schirm, A.L. & Lazar, N.A. 2019. Moving to a world beyond "p< 0.05".***The American Statistician***, 1-19.

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Bayesian Analysis
Estimate Posterior Distributions

- Bayes' Rule

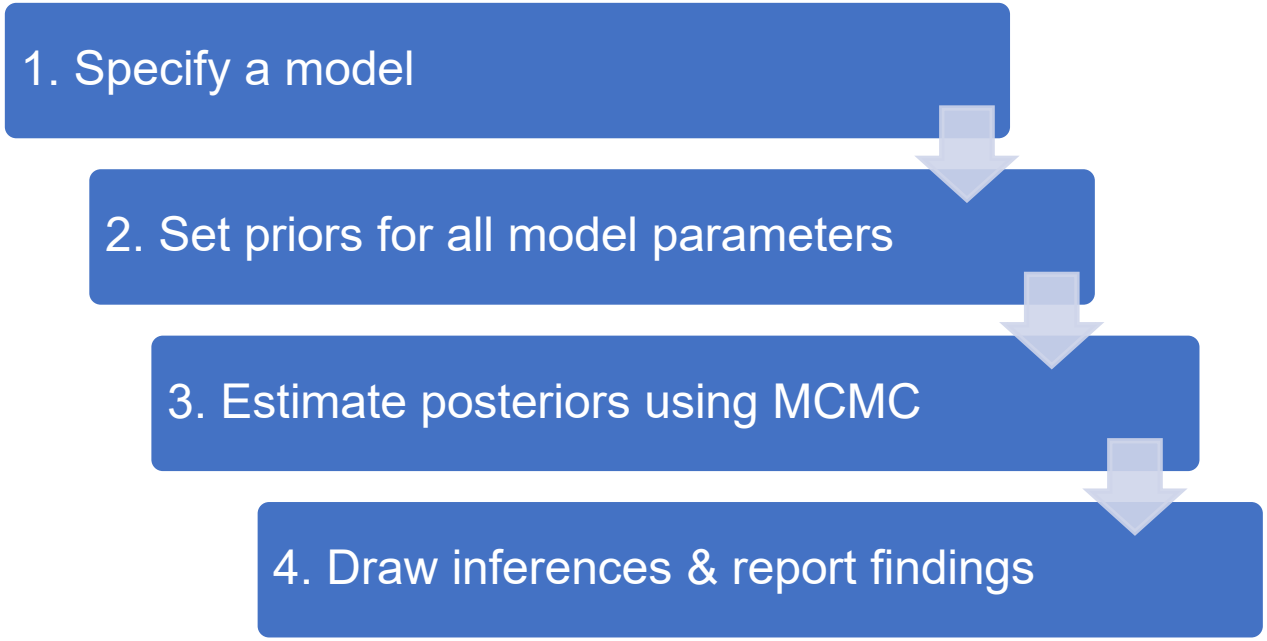
$$p(\theta|\text{data}) \propto p(\text{data}|\theta)p(\theta)$$

$$\text{“posterior”} \propto \text{“likelihood”} \times \text{“prior”}$$

- Bayesian analyses calculate posterior distributions of the proposed effects by updating the expected distribution (prior) using the observed data (likelihood).

Bayesian Analysis:
Estimating Posterior Distributions

Bayesian Modeling workflow



Bayesian Analysis: Estimating Posterior Distributions

- Bayesian statistics offer established methodology for drawing inferences and assessing plausibility of theories.
- Bayesian analysis advantages
  - Predictive probability statements for hypothesized effects based on the specific observed data.
  - Posterior distributions estimate both the size of effects and associated uncertainty.
  - Bayesian updating enables incorporating prior knowledge about hypothesized effects.

Bayesian Studies: An Illustrative Example

Hansen, Perry, & Reese (2004). A Bayesian Operationalization of the Resource-Based View. ***Strategic Management Journal*****.**

Bayesian Analysis
Hansen et al. (2004) Strategic Management Journal

- Research Objective: Effects of administrative decisions on firm performance.
- Hypothesis: Buying organizational units improves firm performance (accounting returns).
- Method:
  - Bayesian hierarchical linear analysis that also accounts for industry, firm and year effects.
  - Markov-Chain Monte Carlo method & Gibbs sampling
  - Flat "uninformative" priors (robustness check for alternative priors)

Bayesian Analysis
Hansen et al. (2004) Strategic Management Journal

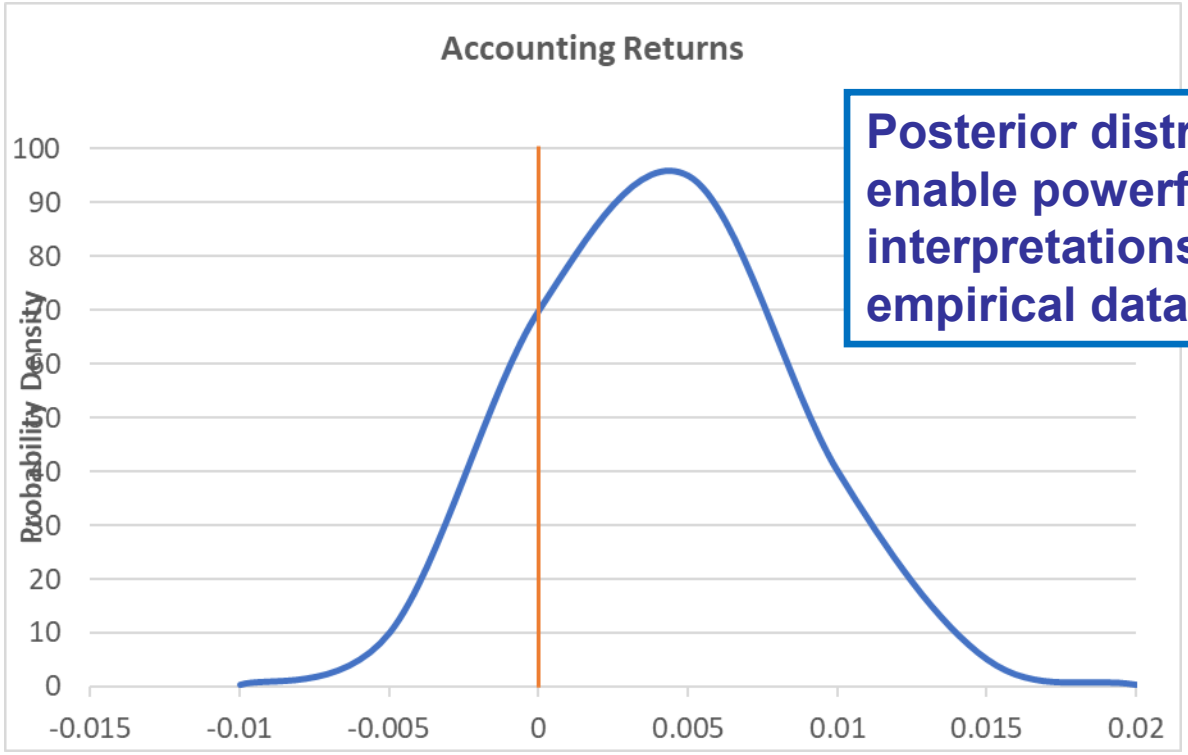
- Results: Probabilities of Effects of Administrative Actions on Firm Performance

Actions	Accounting Returns
Buying units	0.8111
Selling units	0.5584
Org. restructuring	0.9274
Alliances	0.4571
Hiring	0.5867
New Markets	0.4646
Finanical restructuring	0.0516
Personnel changes	0.1169
Lay-offs	0.3082
New products	0.7434

Buying units has 81% probability of improving firm performance.

Bayesian Analysis
Hansen et al. (2004) Strategic Management Journal

- Posterior Distribution of Effects on Firm Performance

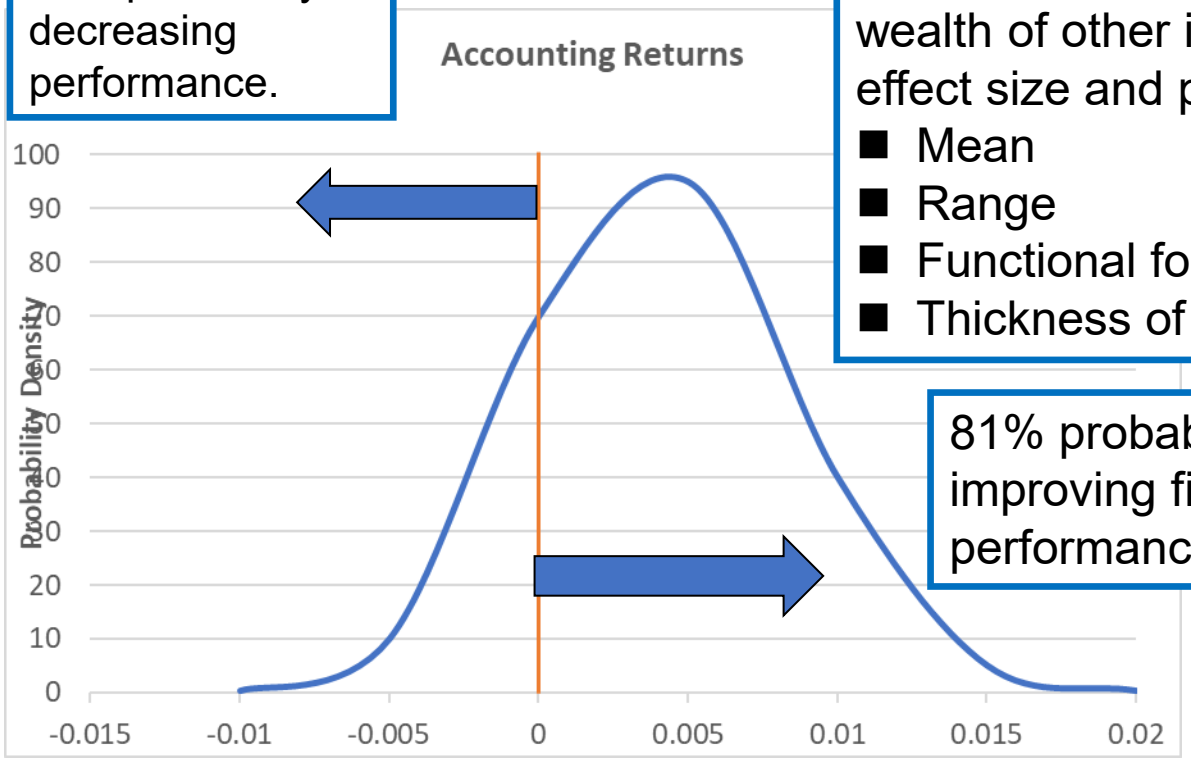


Posterior distributions enable powerful interpretations of the empirical data.

Bayesian Analysis
Hansen Et al. (2004) Strategic Management Journal

- Posterior Distribution of Effects on Firm Performance

19% probability of decreasing performance.



… but this graph also conveys a wealth of other information about effect size and probability.

- Mean
- Range
- Functional form
- Thickness of tails

81% probability of improving firm performance.

... and this can be translated in USD values.

Bayesian Analysis
Model comparisons with Bayes Factors

- Bayes factor estimates the probability that a proposed model fits the data better compared to a baseline model.
- Enables hypothesis test of models with and without the variable of interest – plus, any other model comparisons.
- Dichotomous comparison but quantified probability statements:

  "The odds are 10 to 1 that
  the hypothesized effect improves model fit."

- In most cases, the posterior distributions offer more valuable and detailed information about the hypothesized effect compared to Bayesian factor analysis.

First Conclusions

- Bayesian analyses can be applied to virtually any research question, statistical problem, and empirical context. (McElreath, 2020; Hahn, 2014)
- Bayesian analyses estimate effect sizes and effect uncertainty based on posterior distributions

Next Topics

- Role and specification of prior distributions
- Execution of Bayesian model estimation
- Reporting results and publishing Bayesian studies

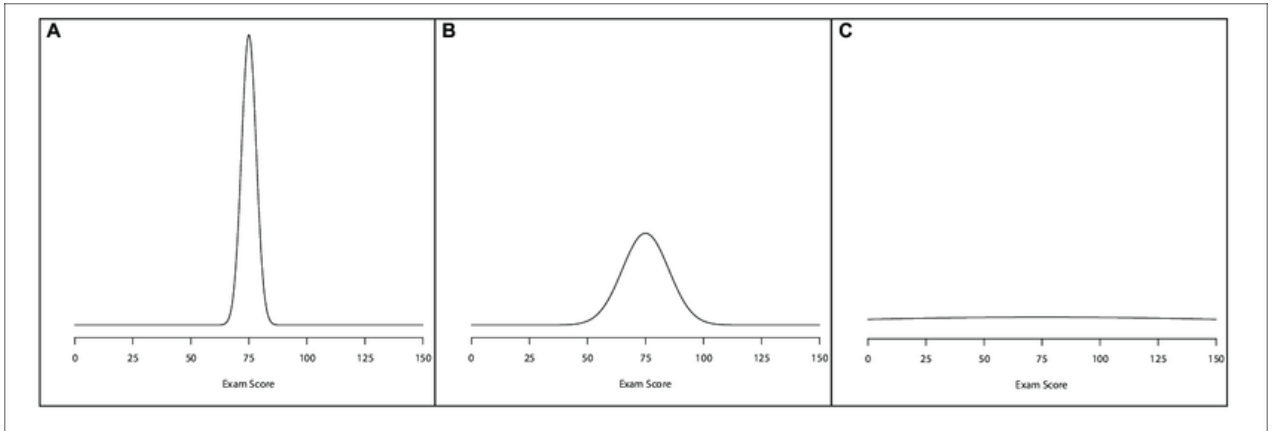
Role and Specification of Prior Distributions in Bayesian Analysis

Role of priors in Bayesian analysis

- What are priors?
  - Prior distributions are predictions about effect distributions.
- What functions do priors serve?
  - Required for Bayesian analysis.
  - Priors are updated using empirical observations.
  - Priors *can* influence results.
  - Priors provide opportunities to account for what we already know about effects.

How to Specify Prior Distributions

- Specify upfront "prior" probability distributions for each variable in the model for its effect on the DV.
  - Specify functional form and hyperparameters
- Levels of informativeness
  - (A) Informative, (B) weakly informative, and (C) diffuse



Bayesian Analysis
Step 1: Specify Prior Distribution

- How to specify a "prior" probability distribution?
- Alternative approaches to specify and justify priors
  - Uninformed priors
  - Priors from previously published empirical studies
  - Priors based on theories
  - Sequential priors
  - Priors from experts and knowledgeable individuals
- Next: Brief outlines of each approach!

[no comprehensive treatment]

(1) Uninformative priors

- Used when researchers nothing or very little about the study phenomenon
- Uniform priors:

  All outcomes are equally likely
- Weakly informative (locally uniform) priors:

  Centered with large assigned variances
- Currently, uninformative priors are frequently used and default in statistical software [easy option]

Limitations of uninformative priors

- Makes strong assumptions that are frequently unrealistic based on what we already know about the phenomenon [e.g., prior research]
- Use as "defaults" encourages ritualized applications instead of careful analyses and adjustments (software)
- Recommendation:
  - Use only when very little or no prior knowledge
  - Justify distrust in existing evidence and previous research
  - Use in cases of data dominance
  - Useful benchmark to evaluate impact of "other" priors

(2) Priors from published empirical research

- Use results of available empirical studies to estimate priors (= accumulation of knowledge across studies)
- Potential challenges
  - Current lack of exact replications
  - Dissimilarity of prior related studies
  - Trust in prior studies (e.g., replication crisis)
- <u>Recommendation:</u>
  - Aggregate from a few carefully selected studies
  - Meta analyses (Aguinis et al., 2011)

(3) Priors based on theories

- Feasible for quantified theories that provide for uncertainty
- Recommendation:
  - Powerful to incorporate established knowledge
  - … but translating abstract and vague management theories into specific outcome distributions often challenging

(4) Sequential priors

- Priors based on data collected in an earlier stage of a sequential data-collection process
- Relevant approach in important emerging empirical contexts
  - Big data
  - Continuous data collections
- Recommendation:
  - Powerful in iterative and multi-stage research designs

(5) Elicit priors from experts and lay people

- Priors based on the intuitions of experts or knowledgeable lay people, not the researchers
- Challenges of capturing individuals' intuitions
  - Recruiting individuals
  - Individuals' perception biases
  - Translating perceptions into probability distributions
- Recommendation:
  - Training of subjects
  - Use graphic computer-based systems
  - Emerging advanced elicitation tools

First Conclusion

- Priors are important opportunities for reflective empirical analyses accounting for uncertainty
  - Focus researchers on distributions of effects
  - Encourage explicit justification of priors selected
  - Transparency protects against "arbitrary" or "outcome-driven" choice of priors

Second Conclusion

- Established alternative approaches to constructing priors are available
  - Priors provide opportunities to integrate knowledge from prior empirical research
  - If multiple approaches are feasible this enables comparing and combining the priors each generates.

Third Conclusion

- Always conduct sensitivity analyses for the impact of priors on results
  - The larger the sample, the more data dominate and priors have limited effects
  - The smaller the sample, the stronger the effect of priors on the posterior
- Trust-worthy priors can enable the meaningful empirical investigation of small samples

General Conclusions

- Bayesian analyses foster scientific progress by providing a systematic way to integrate what we already know into analyses of new data.
- **"Priors" are an opportunity!**
- … interpreting data using carefully chosen priors promises more accurate posterior distributions and stronger contributions to theory development.
- **"Priors" are an advantageous Bayesian feature!**

Execution of Bayesian Model Estimation

Execution of Bayesian Estimations

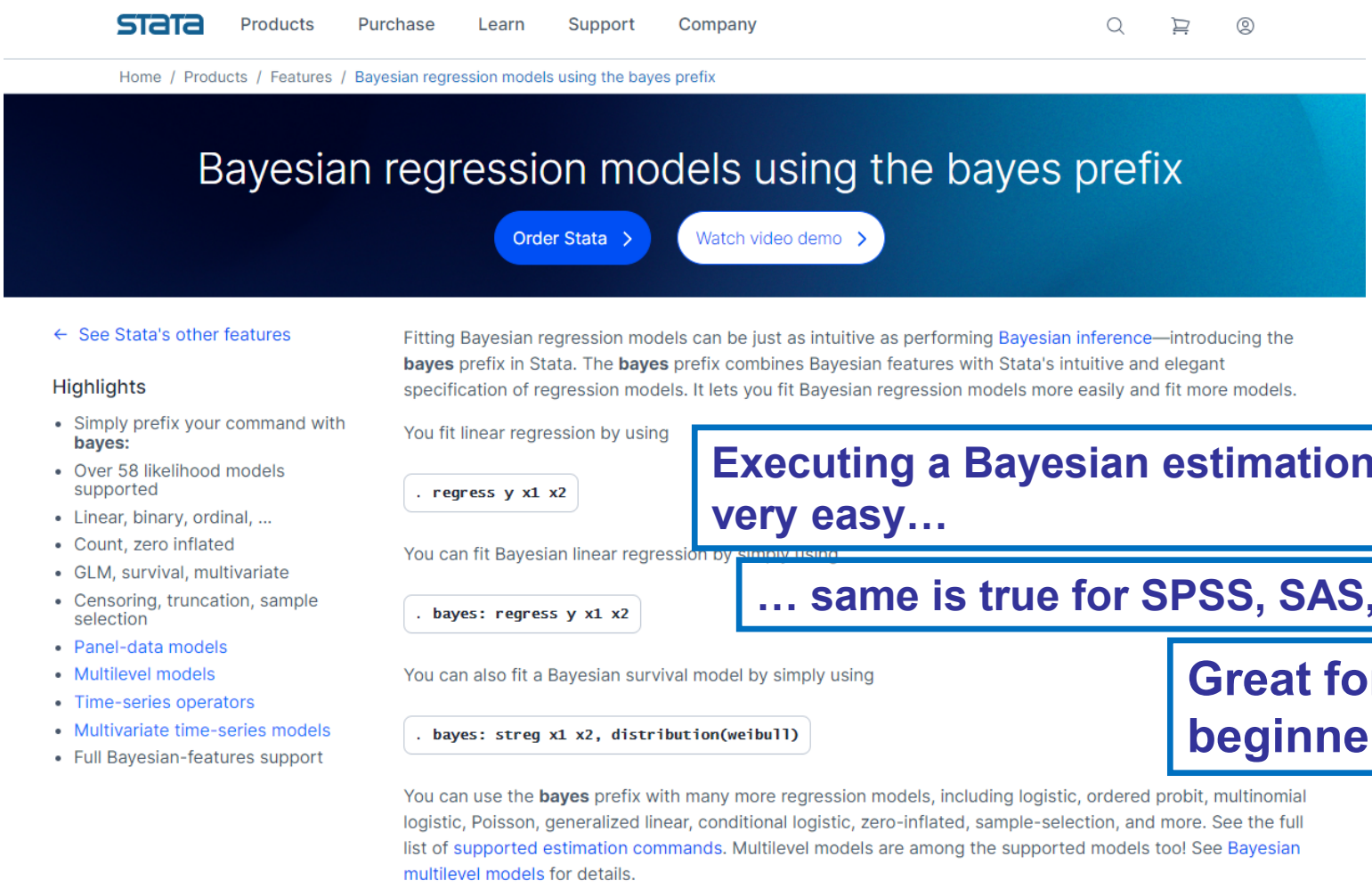
- In the past, the execution of Bayesian estimations was cumbersome and tedious which constrained applications for centuries.
  - Algebraic estimations of posterior distributions required "conjugate" priors (or luck) and substantial mathematical skills
  - Selecting conjugate priors for algebraic convenience raised legitimacy concerns
- Today's advanced computer technology facilitates Bayesian estimations.
  - Advanced algorithms using Markov-Chain Monte-Carlo simulations estimate posterior for any type of priors.
  - All common statistical software packages run Bayesian analyses.

Range of Software Solutions: From Canned to Customized

Canned Routines

Code your Own

Bayesian Analysis in STATA



Executing a Bayesian estimation is very easy…

… same is true for SPSS, SAS, others

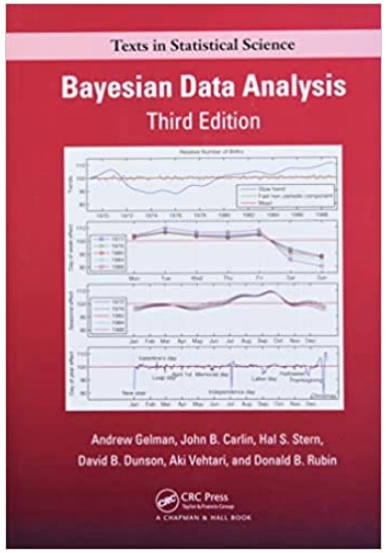
Great for beginners!

Bayesian Analyses in R with Stan

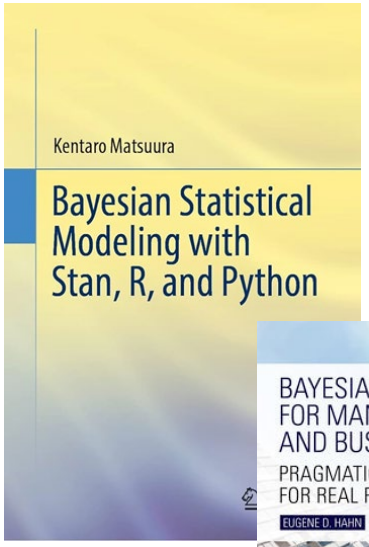
Great for intermediate to advanced users

How to learn more about Bayesian analysis?

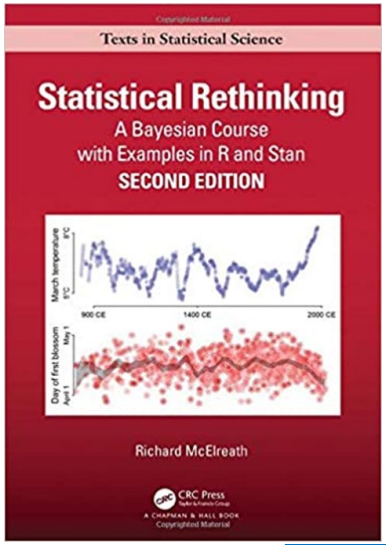
- Plenty of resources



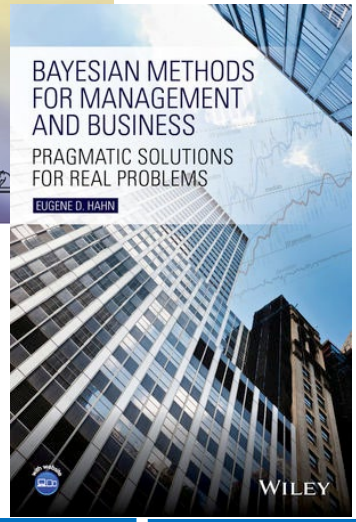
Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. (2014). Bayesian Data Analysis.



Kentaro Matsuura (2022). Bayesian Statistical Modeling with Stan, R, and Python



Richard McElreath (2020). Statistical Rethinking: A Bayesian Course with Examples in R and Stan [Textbook, Lecture Series]



Eugene Hahn (2014). Bayesian Methods for Management and Business

Bayesian stats courses at most universities …

.. and find a Bayesian-trained collaborator!

Summary:
Execution of Bayesian Estimations

- Multiple great software tools available for conducting Bayesian inference
- Beginners should start fitting simple models using familiar software platforms (e.g., Stata, SAS, R)
- For more complicated models, consider using flexible probabilistic programming platforms like Stan.

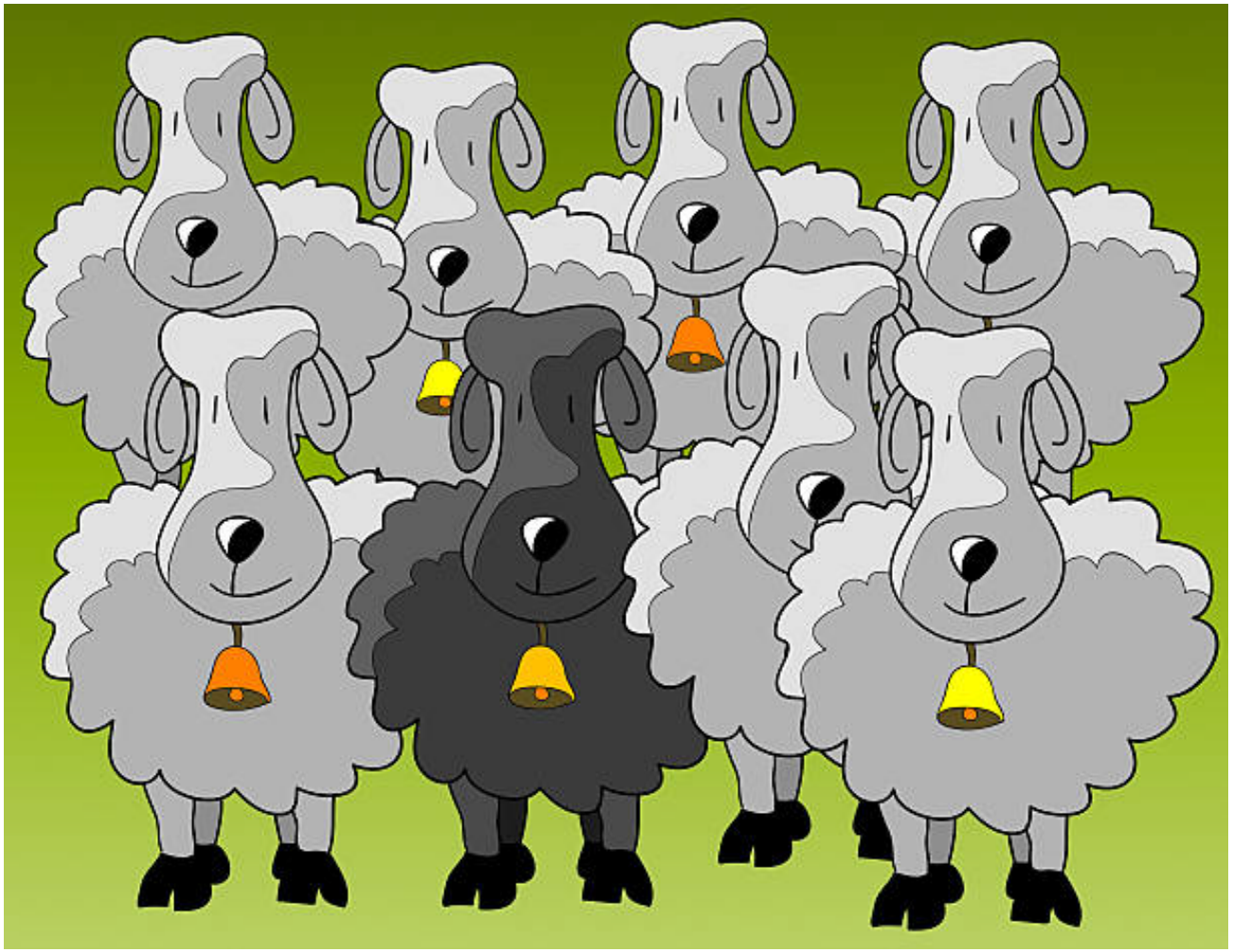
How to Report and Publish Bayesian Studies in Management Journals

Bayesian studies disappoint expectations

Editors, reviewers, and readers expect:

1. Statistical significance tests.
2. Null hypotheses, not priors.
3. Very simple results – statistically significant or not?

Bayesian studies are different



Framing a Bayesian Study

- Anticipate a skeptical audience (editors, reviewers, readers).
- Explain and highlight the benefits of Bayesian analysis …
- …without attacking frequentist methods directly.
- Intro and hypothesis development are similar and still critical.
- Methods, results, and interpretation of results fundamentally differ.
- Expect some but not all your reviewers to be familiar with Bayesian analysis.

How to Report Results

- Reviewers will likely ask for comparisons of Bayesian to Frequentist results.
- Show graphs of posterior distributions and interpret them -- in addition, provide tables with parameter and interval estimates.
- Go beyond dichotomous “effect/no effect” interpretations of the empirical data
- Go beyond “averages” and discuss effect size distributions.
- The main output is the posterior distribution!!

Explain that a prior distribution is not a null hypothesis

- Null hypotheses pay no attention to previous research or to knowledge about the phenomena.
- Priors are starting points for data analysis that can take advantage of previous research.
- Always perform analysis to communicate the sensitivity of results to different prior specifications.

Summary:
Bayesian analysis in management research

Summary:
Bayesian analysis in management research

- Bayesian analysis can be applied to virtually any research question, statistical problem, and empirical context.
- Use of prior distributions implies opportunities for building on prior research and better accumulation of knowledge across studies.
- Advancements in computer hardware and software have enabled Markov-Chain Monte-Carlo simulation-based efficient Bayesian model estimation.
- Publication of Bayesian studies in top journals is feasible and is increasingly facing better odds.
- What is needed is a willingness of management scholars to become familiar with this different but powerful statistical paradigm!

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